Routing and wavelength assignment in optical networks

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Optical networks

- Links: optical fibers
- Wavelength Division Multiplexing (WDM)
  - several ``channels´´ per fiber
- Routing
- Wavelength assignment

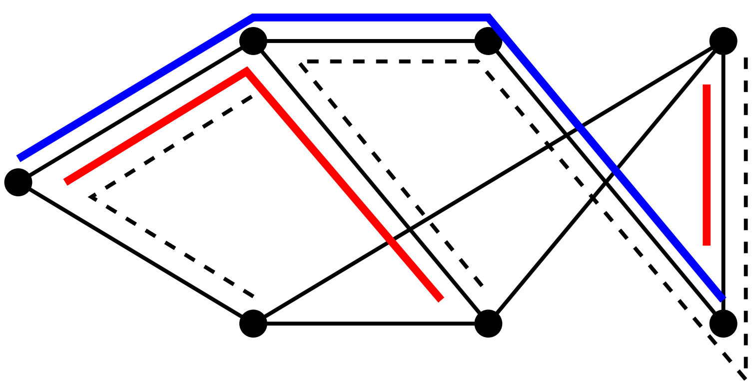
Considerations in WDM networks

- Restrictions:
  - requests using the same fiber must use different wavelengths (colors)
  - wavelength continuity
- Limited availability of wavelengths

Maximum profit path coloring

Def. MaxPr-PC problem:

- instance: graph G, path set $\mathcal{P}$, profits $w : \mathcal{P} \to \mathbb{Q}$, # colors k
- solution: a k-colorable subset of paths $\mathcal{P}' \subseteq \mathcal{P}$
- goal: maximize $w(\mathcal{P}') = \sum_{p \in P'} w(p)$



Outline of presentation

- Algorithms for MaxPr-PC in rings and experimental evaluation
- Non-cooperative routing and wavelength assignment in multifiber optical networks
- A neat application of path coloring to a transportation problem
- Conclusions

Maximum profit path coloring

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Remarks:

- NP-hard in rings and trees
- polynomial-time solvable in chains (CL95)

Related work

- MaxPr-PC with routing (BG09), $\rho = 1.5$
- MaxPr-PC with routing and capacity constraints (LLWZ05), $\rho = 2$
- Adaptation of iterative algorithm (WL98), $\rho \approx 1.58$
- LP + randomized rounding (Car07), w.h.p. $\rho \approx 1.49 + \epsilon$

Coming up...

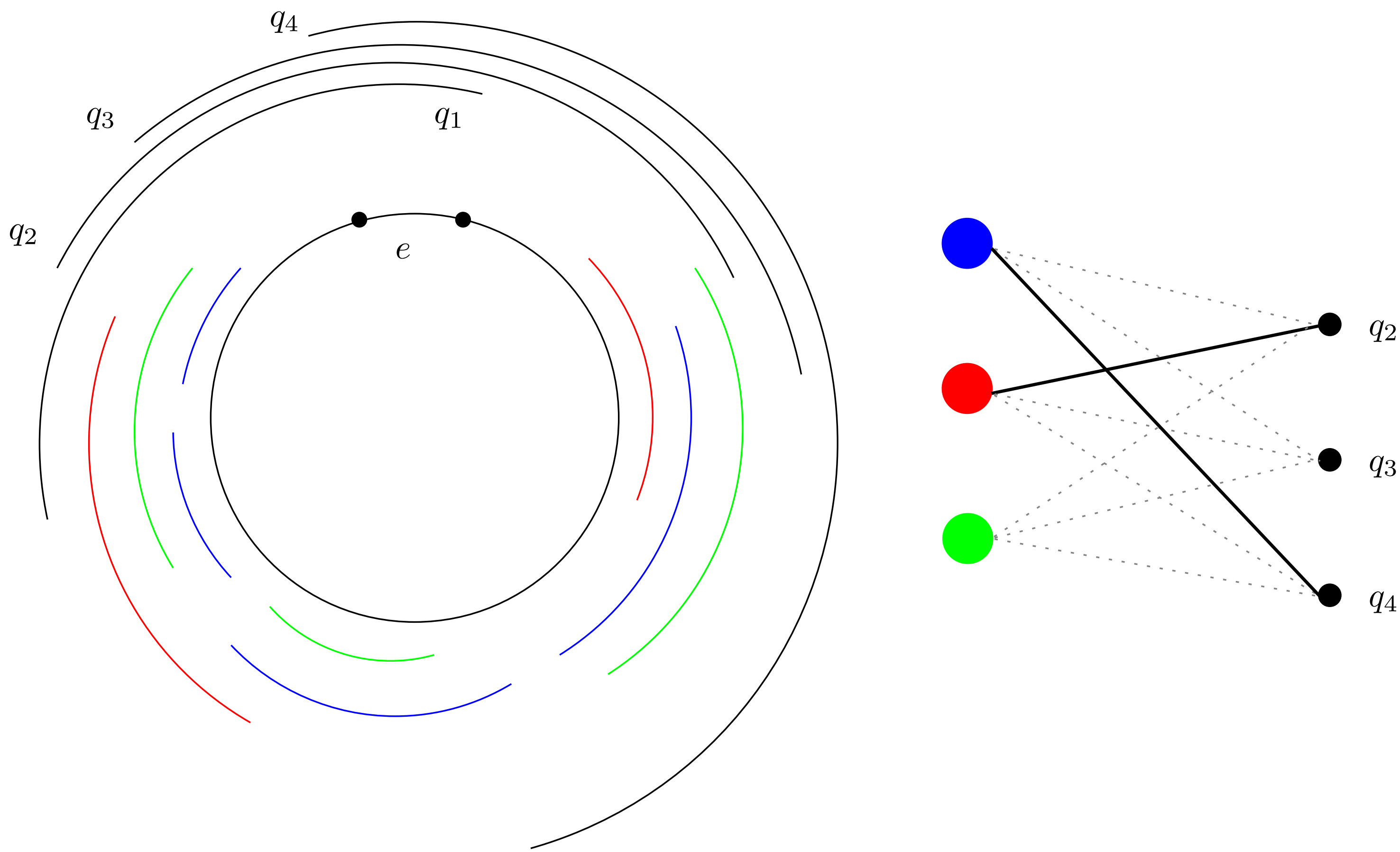
- Match and replace
  - a fast, combinatorial 2-approximation algorithm for MaxPr-PC in rings
- Tradeoffs between time efficiency and attained profit
  - some experimental results

B, Pagourtzis, Potika: Maximum profit wavelength assignment in WDM rings. CTW 2008 (full version to appear in *Networks*).

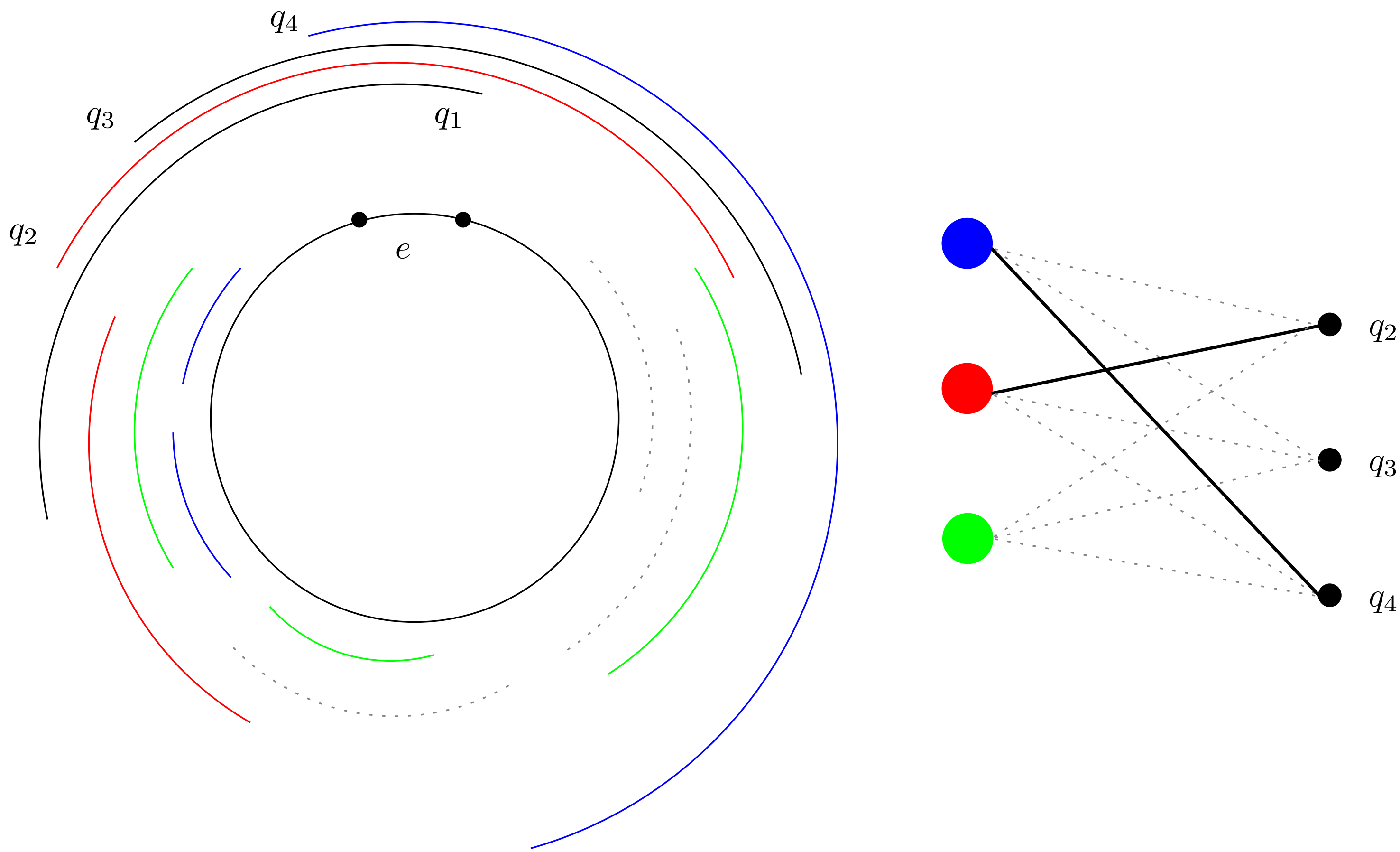
Match and replace

1. pick an edge e, and partition $\mathcal{P}$ into $\mathcal{P}_e$ and $\mathcal{P}_c$
2. color $\langle G, \mathcal{P}_c, w, k \rangle$ optimally (chain subinstance)
3. construct a weighted complete bipartite graph H with nodes $\{1, \ldots, k\} \cup K$ (K: set of k heaviest paths in P_e)
   - $w'(i, q) = w(q) - w([\mathcal{P}_c(i)]^q)$ (gain by picking $q \in \mathcal{P}_e$ instead of $[\mathcal{P}_c(i)]^q$)
4. compute a maximum weight matching M in H
5. **for each** $(i, q) \in M$
6. uncolor all paths in $[\mathcal{P}_c(i)]^q$ and color q with i

Match and replace (cont'd)



Match and replace (cont'd)



Match and replace (cont'd)

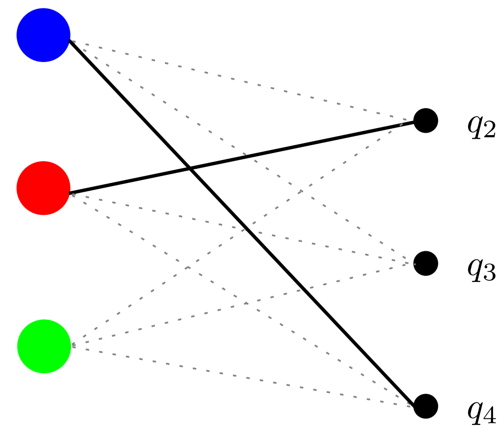
- $\mathrm{OPT} \leq \mathrm{OPT}_e + \mathrm{OPT}_c$
- We need to show that
  - $\mathrm{OPT}_c \leq \mathrm{SOL}$ (easy!)
  - $\mathrm{OPT}_e \leq \mathrm{SOL}$
- These imply $\mathrm{OPT} \leq 2\mathrm{SOL}$
- Remains to prove:

$$\mathrm{OPT}_e = w(K) \leq \mathrm{SOL}$$

Match and replace (cont'd)

The solution returned has total profit

$$\mathrm{SOL} = \mathrm{SOL}_c + w'(M) \ ,$$

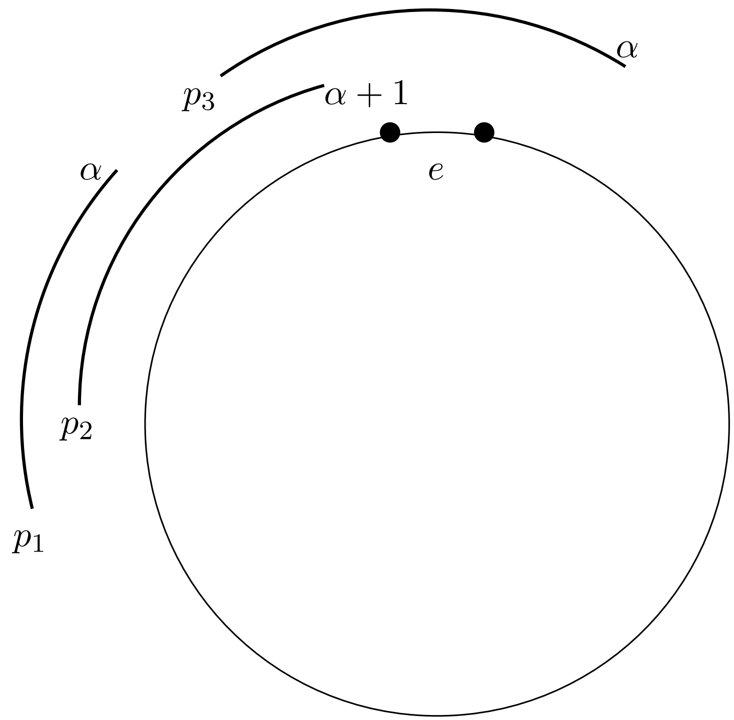


which can be written as

$$\mathrm{SOL} = \sum_{i \text{ not matched}} w(\mathcal{P}_c(i)) + w(K_M) + \sum_{(i,q)\in M} w([\mathcal{P}_c(i)]^{\neg q})$$

$$\geq \sum_{i \text{ not matched}} w(\mathcal{P}_c(i)) - \sum_{q \text{ not matched}} w(q) + w(K) \geq \mathrm{OPT}_e \ .$$

Tight example



- Only one color
- $\frac{\text{OPT}}{\text{SOL}} = \frac{2a}{a+1} \longrightarrow 2$

Alternatives for MaxPr-PC in rings

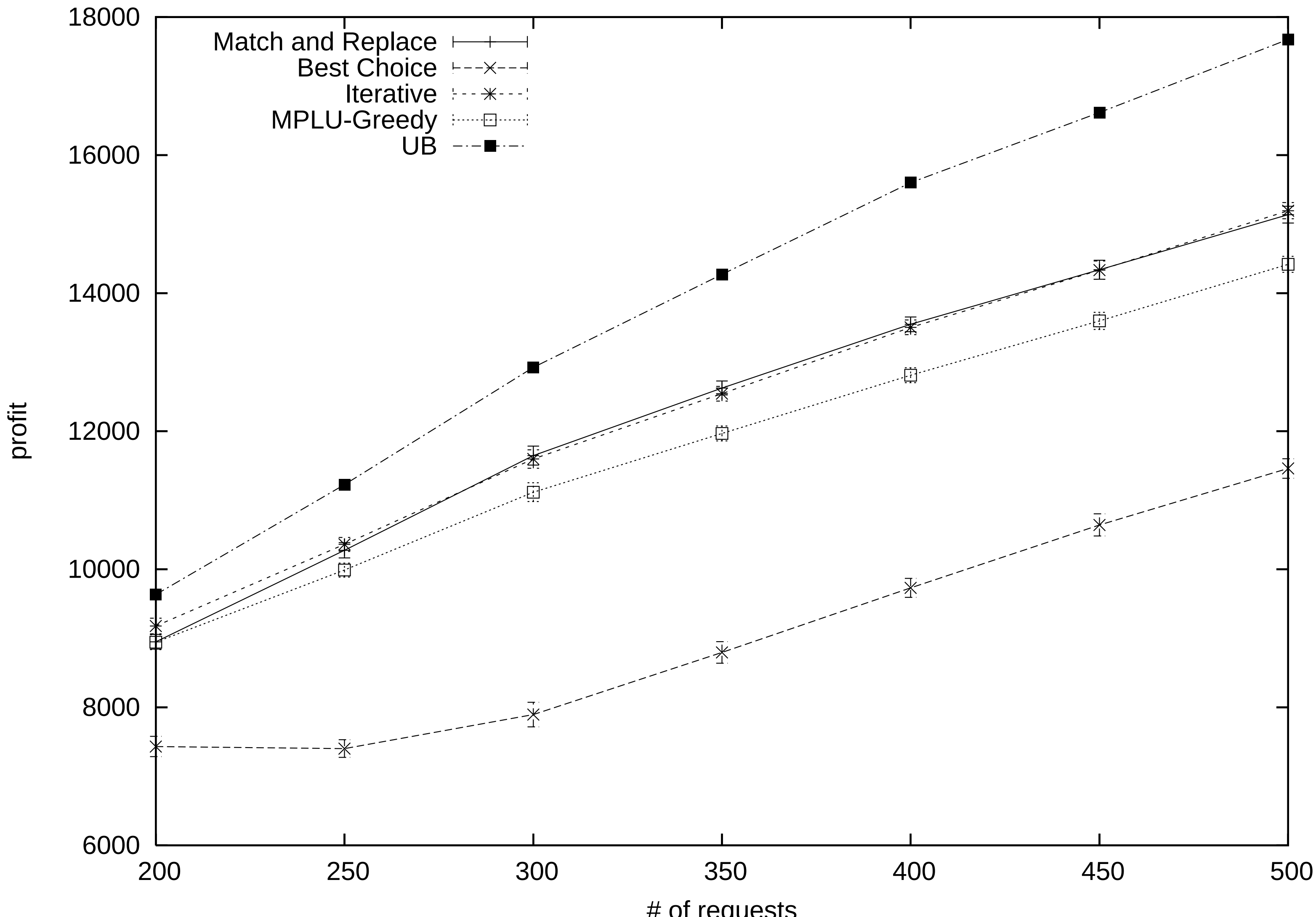
Algorithm	Running time	Appr. guarantee
Iterative	$\mathcal{O}(k^2 m^2 \log m)$	1.58
M&R	$\mathcal{O}(m^2(k + \log m))$	2
Greedy	$\mathcal{O}(nmk + m \log m)$	non-constant
``Best''	$\mathcal{O}(km \log m)$	2

Experimental results: instance packs

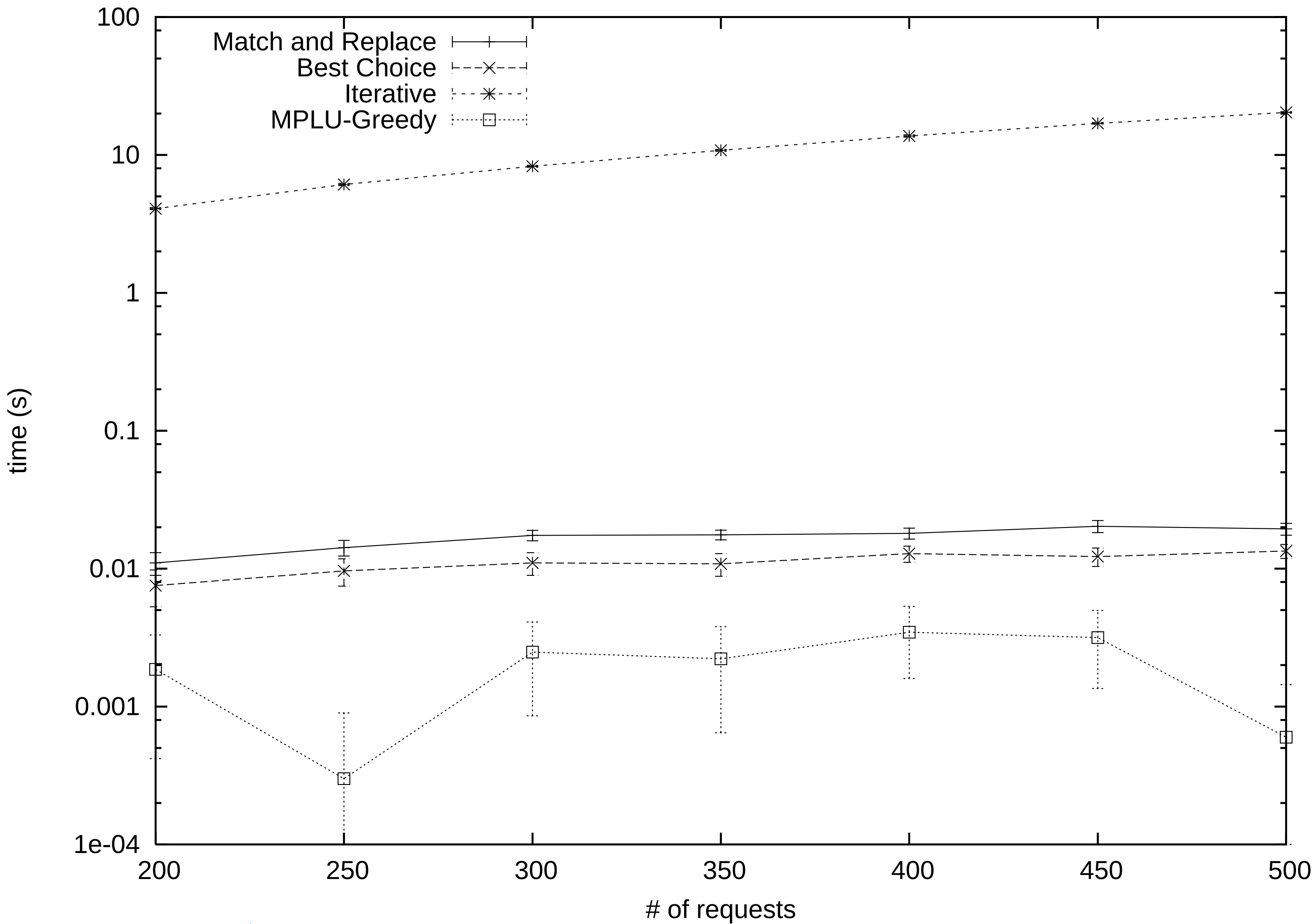
50 randomly generated instances w.r.t. the following parameters:

- n: # nodes
- m: # requests
- k: # colors
- W: upper bound on profits
- mode of generation: uniform or gaussian:μ:σ

profit vs. # paths (n=100, k=80)



time vs. # paths (n=100, k=80)



Ranking of algorithms

Algorithm	Attained profit	Approximation ratio
Iterative	★★★★	1.58
Match-and-Replace	★★★★	2
Greedy	★★★	non-constant
Best-Choice	★	2

Algorithm	Time efficiency	Time complexity
Greedy	★★★★	$\mathcal{O}(nmk)$
Match-and-Replace	★★★	$\mathcal{O}\left(m^2(k + \log m)\right)$
Best-Choice	★★★	$\mathcal{O}(km \log m)$
Iterative	★	$\mathcal{O}\left(k^2 m^2 \log m\right)$

Cardinality version

- All requests have the same weight
- Routed/unrouted requests
- Experimental results of the same flavor: iterative is best, closely followed by ``Combine'', simple greedy algorithm performs competently

B, Pagourtzis, Potika: Maximum request satisfaction in WDM rings: Algorithms and experiments. PCI 2007.

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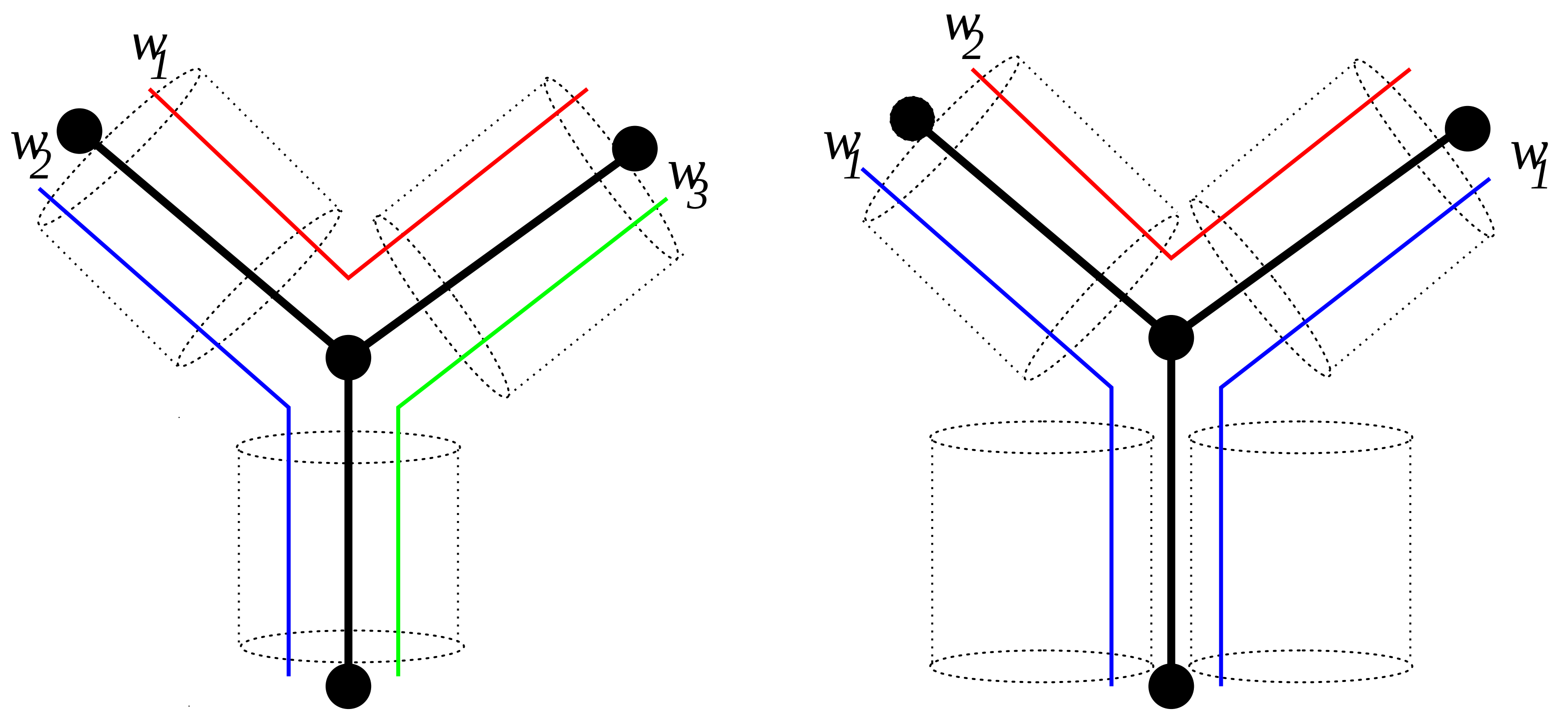
Single *vs* multi-fiber



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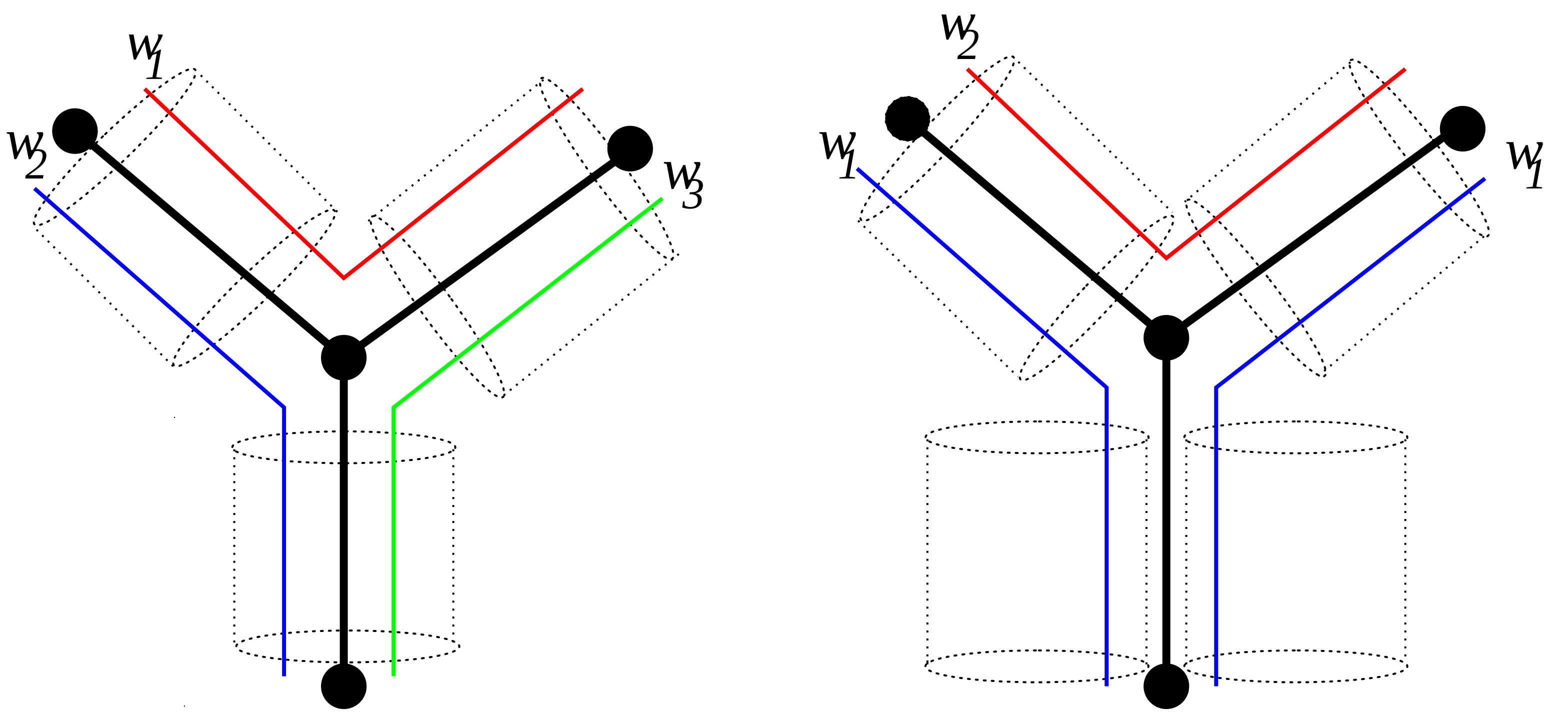

small color multiplicity $\Longrightarrow$ small # fibers

Non-cooperative model

- Large-scale networks: shortage of centralized control
  - provide incentives for users to work for the social good
- Social good: minimize fiber multiplicity
- Reasonable policy: charge users according to the maximum fiber multiplicity incurred by their choice of frequency

B, Pagourtzis, Pierrakos, Potika: On a non-cooperative model for wavelength assignment in multifiber optical networks. ISAAC 2008 (LNCS vol. 5369).

Non-cooperative model

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- Reasonable policy: charge users according to the maximum fiber multiplicity incurred by their choice of frequency

What will be the impact on social welfare if we allow users to act freely and selfishly?

B, Pagourtzis, Pierrakos, Potika: On a non-cooperative model for wavelength assignment in multifiber optical networks. ISAAC 2008 (LNCS vol. 5369).

Problem formulation

Def. PATH MULTICOLORING problem:

- input: graph $G(V, E)$, path set $\mathcal{P}$, # colors k
- solution: a coloring $c : \mathcal{P} \rightarrow W$, $W = \{a_1, \ldots, a_k\}$
- goal: minimize the maximum color multiplicity

$$\mu_{\max} \triangleq \max_{e \in E, a \in W} \mu(e, a)$$

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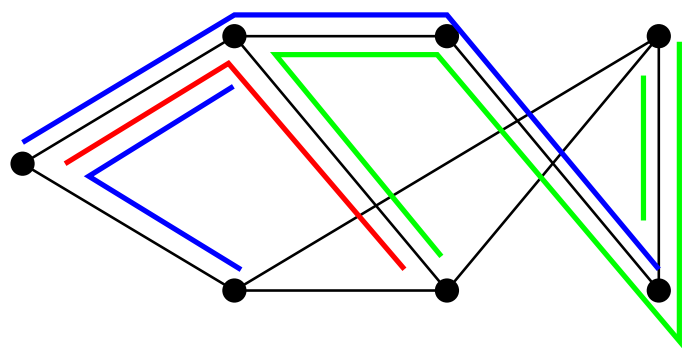

$L = 3$

$\mu_{\max} = 2$

$\mu_{\mathrm{OPT}} \geq \left\lceil \frac{L}{k} \right\rceil$

Game-theoretic formulation

- Def. Given a graph G, path set $\mathcal{P}$ and k, define the game $\langle G, \mathcal{P}, k \rangle$:
  - players: $p_1, \ldots, p_{|\mathcal{P}|} \in \mathcal{P}$
  - strategies: each p_i picks a color $c_i \in W$
  - strategy profile: a vector $\vec{c} = (c_1, \ldots, c_{|\mathcal{P}|})$
  - disutility functions: $f_i(\vec{c}) = \mu(p_i, c_i)$ (maximum multiplicity of c_i along p_i)
  - social cost: $\mathrm{sc}(\vec{c}) \triangleq \mu_{\max} = \max_{e \in E, a \in W} \mu(e, a)$

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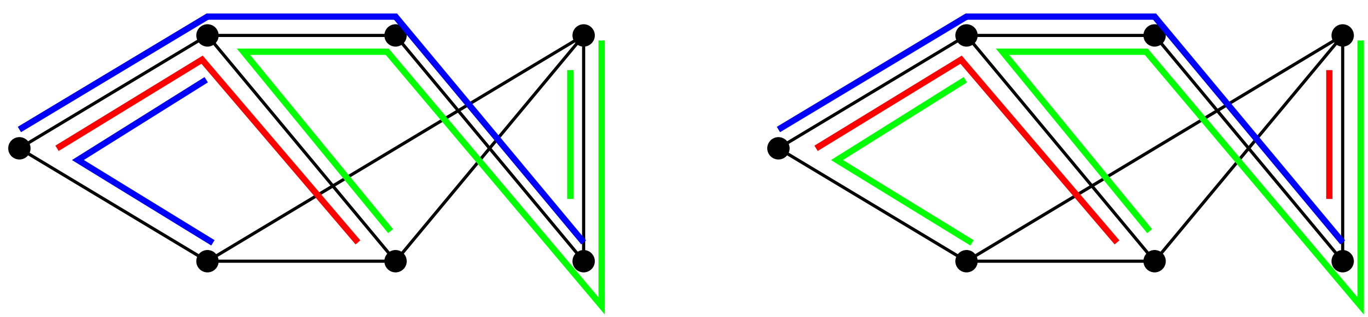
- Def. S-PMC: the class of all $\langle G, \mathcal{P}, k \rangle$ games

Nash equilibria

- Def. A strategy profile is a Nash equilibrium (NE) if no player can reduce her disutility by changing strategy unilaterally:

$$\forall p_i \in \mathcal{P}, \forall c_i' \in W : f_i(\vec{c}_{-i}; c_i) \leq f_i(\vec{c}_{-i}; c_i')$$



Efficiency of Nash equilibria

- **Def.** The price of anarchy (PoA) of an S-PMC game:

$$\text{PoA} = \frac{\max_{\vec{c} \text{ is NE}} \text{sc}(\vec{c})}{\mu_{\text{OPT}}} \triangleq \frac{\hat{\mu}}{\mu_{\text{OPT}}}$$

- **Def.** The price of stability (PoS) of an S-PMC game:

$$\text{PoS} = \frac{\min_{\vec{c} \text{ is NE}} \text{sc}(\vec{c})}{\mu_{\text{OPT}}}$$

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- Rate of convergence to some NE?
  - by repeatedly changing some player's strategy to improve her disutility (Nash dynamics)

Coming up...

- Convergence of Nash dynamics
- Efficient computation of Nash equilibria
- Upper and lower bounds for the price of anarchy
- The price of anarchy on graphs of degree 2

Related work

- Minimization problem with the $\mu_{\max}$ objective (AZ04)
- Minimization problem with the $\sum_{e\in E}\max_{a\in W}\mu(e,a)$ objective (NPZ01)
- Bottleneck network games
  - player cost: MAX of delays along her path
  - players pick among several possible routings (BM06)
  - latency functions on edges (BO06)
- Congestion games (MS96, Ros73)

Convergence to NE

Thm. Any Nash dynamics converges in at most $4^{|\mathcal{P}|}$ steps

- consider the vector

$$(d_L(\vec{c}), d_{L-1}(\vec{c}), \ldots, d_1(\vec{c}))$$

- lexicographic-order argument (attributed to Mehlhorn in (FKK+02))
- $\text{PoS} = 1$

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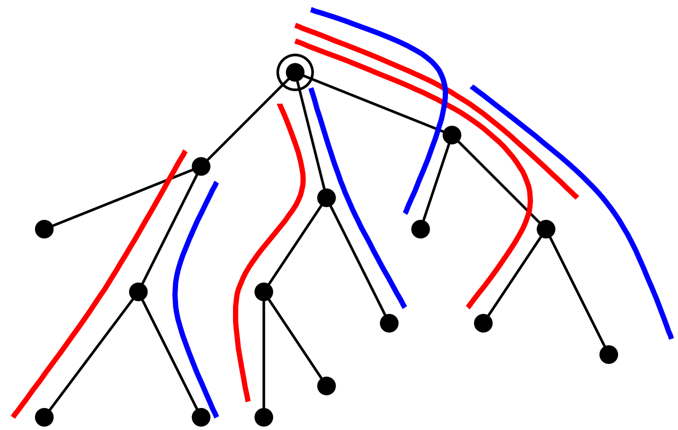
- PoS = 1
- how many such vectors?

$$\binom{|\mathcal{P}| + L - 1}{|\mathcal{P}|} \leq 2^{|\mathcal{P}|+L-1} < 4^{|\mathcal{P}|}$$

Efficient computation of optimal NE



- $\langle G, \mathcal{P}, k\rangle$ is in S-PMC(ROOTED-TREE) if $\exists r$ s.t. each path in $\mathcal{P}$ lies entirely on some simple path from r to a leaf
- consider edges in BFS order: color paths with min-multiplicity color in the partial solution

A simple upper bound on the PoA

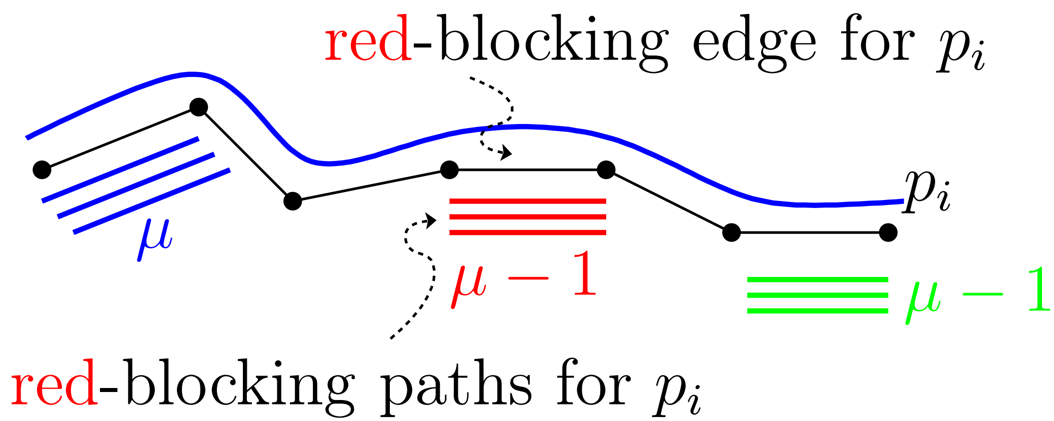
Thm. $\text{PoA}(\langle G, \mathcal{P}, k\rangle) \leq k$

Proof.

$$\text{PoA} = \frac{\hat{\mu}}{\mu_{\text{OPT}}} \leq \frac{\hat{\mu}k}{L} \leq k$$

A structural property of NE

- If $\vec{c}$ is a NE, then for any $p_i \in \mathcal{P}$ and for any $a \in W$ there is an $e \in p_i$ s.t. $\mu(e, a) \geq f_i(\vec{c}) - 1$



An upper bound on the PoA

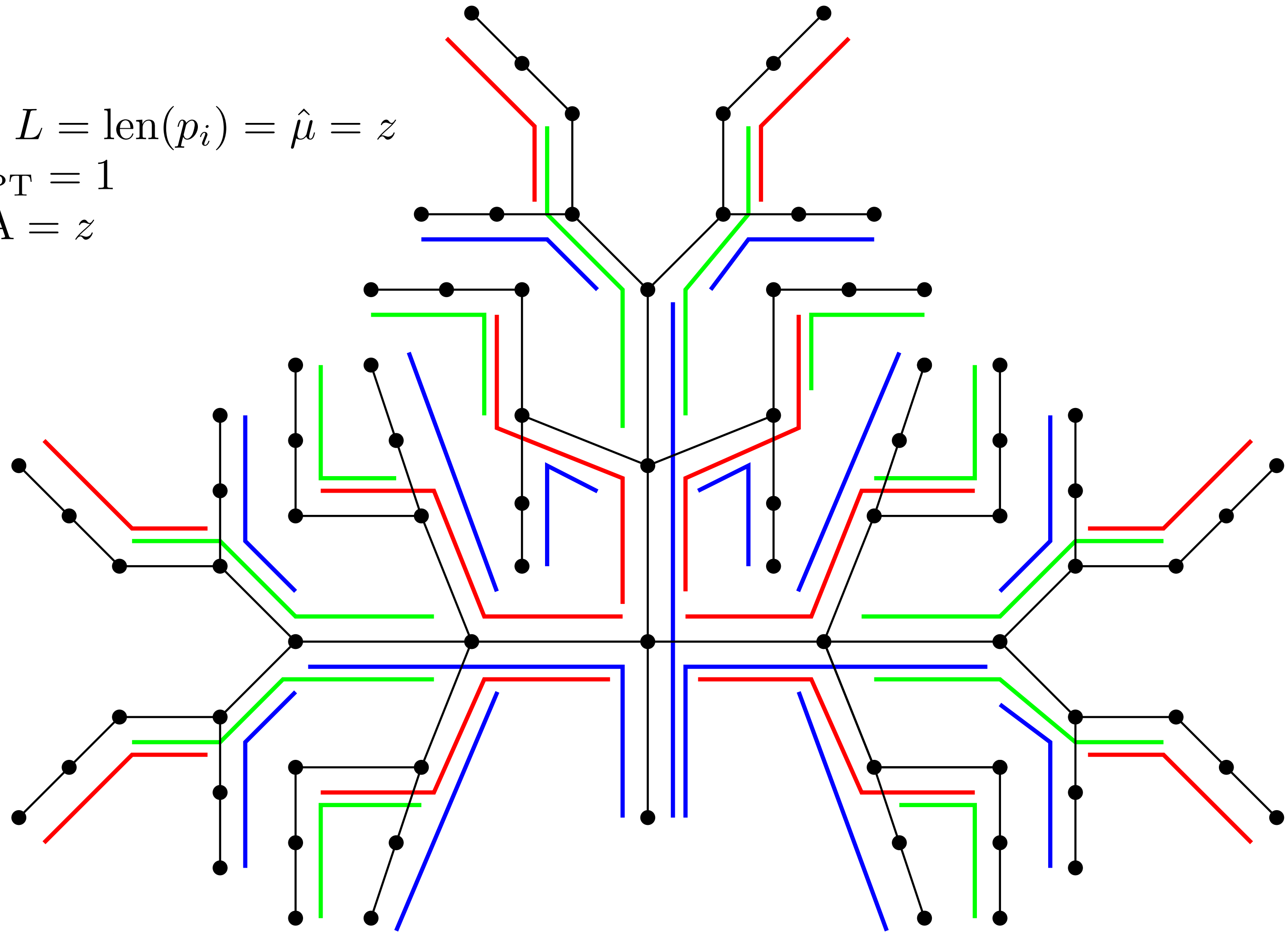
Thm. If $\vec{c}$ is a NE and $\mathrm{sc}(\vec{c}) = f_i(\vec{c}) = \hat{\mu}$ then $\mathrm{PoA} \leq \mathrm{len}(p_i)$

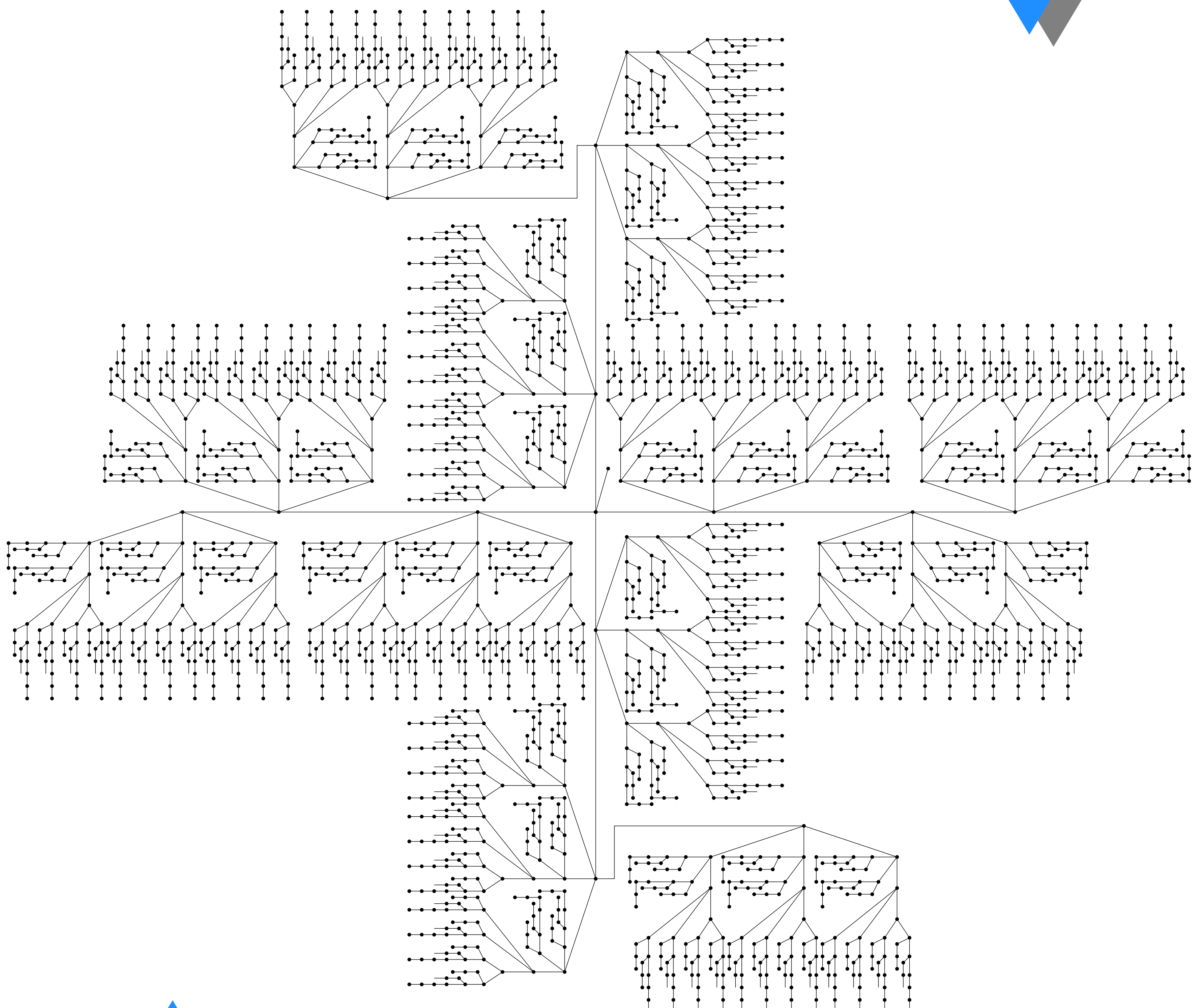
Proof.

- all k colors are blocked along p_i
- some edge of p_i must block at least $\left\lceil \frac{k}{\mathrm{len}(p_i)} \right\rceil$ colors
- max load is $L \geq 1 + \left\lceil \frac{k}{\mathrm{len}(p_i)} \right\rceil (\hat{\mu} - 1)$
- $\mu_{\mathrm{OPT}} \geq \left\lceil \frac{L}{k} \right\rceil$
- $\mathrm{PoA} = \frac{\hat{\mu}}{\mu_{\mathrm{OPT}}} \leq \frac{\hat{\mu}}{\left\lceil \frac{1+\left\lceil \frac{k}{\mathrm{len}(p_i)} \right\rceil (\hat{\mu}-1)}{k} \right\rceil} \leq \mathrm{len}(p_i)$

A matching lower bound

$k = L = \text{len}(p_i) = \hat{\mu} = z$
$\mu_{\text{OPT}} = 1$
$\text{PoA} = z$





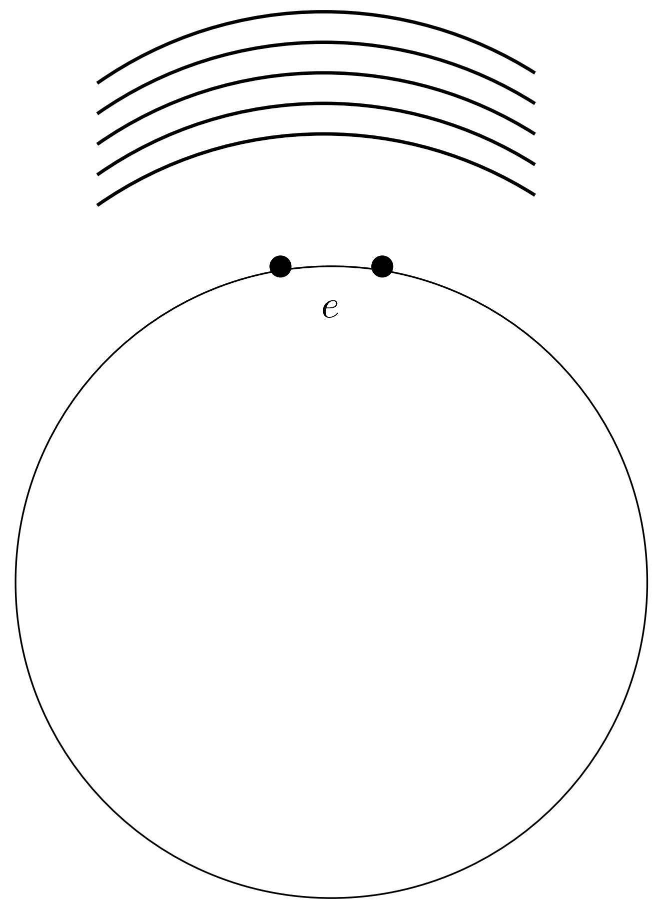
What about graphs with degree 2?

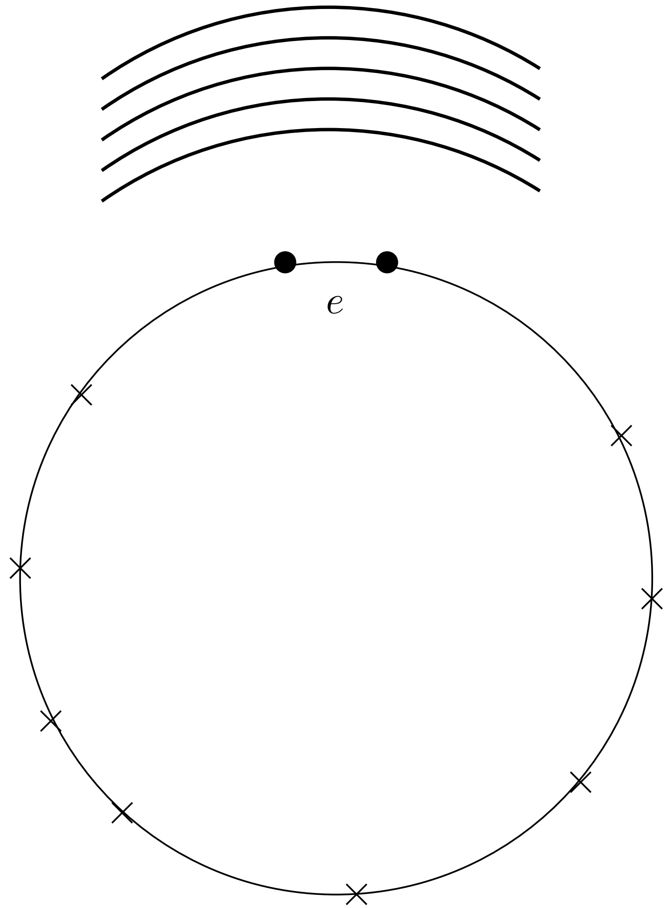
A more involved structural property:

$P(e, a_i)$: the set of paths using edge e that are colored with a_i.

Lem. In any NE of an S-PMC(RING) game, $\forall$ edge e and $\forall a_i$ there is an arc s.t.:

- $\forall a_j \neq a_i$ the arc contains an edge which is an a_j-blocking edge for at least half of the paths in $P(e, a_i)$, and
- $\forall e'$ in the arc, $|P(e', a_i) \cap P(e, a_i)| \geq \left\lceil \frac{|P(e,a_i)|}{2} \right\rceil$


e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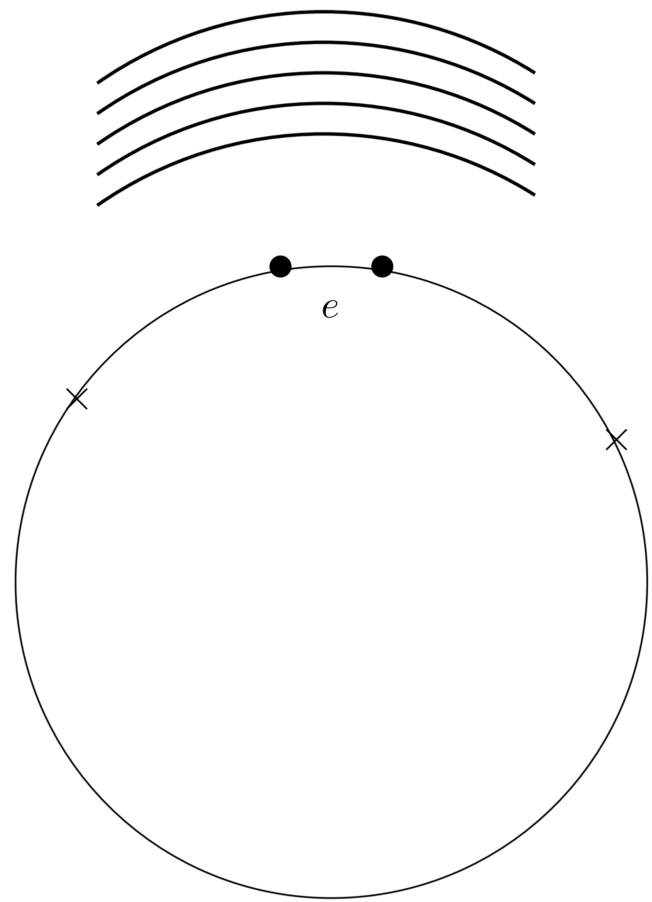
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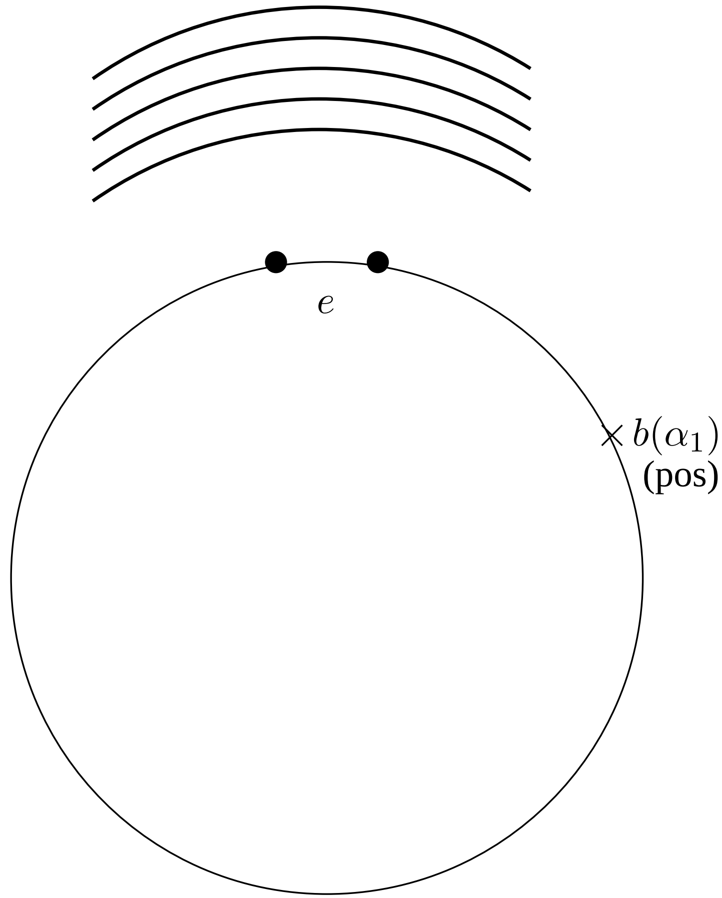
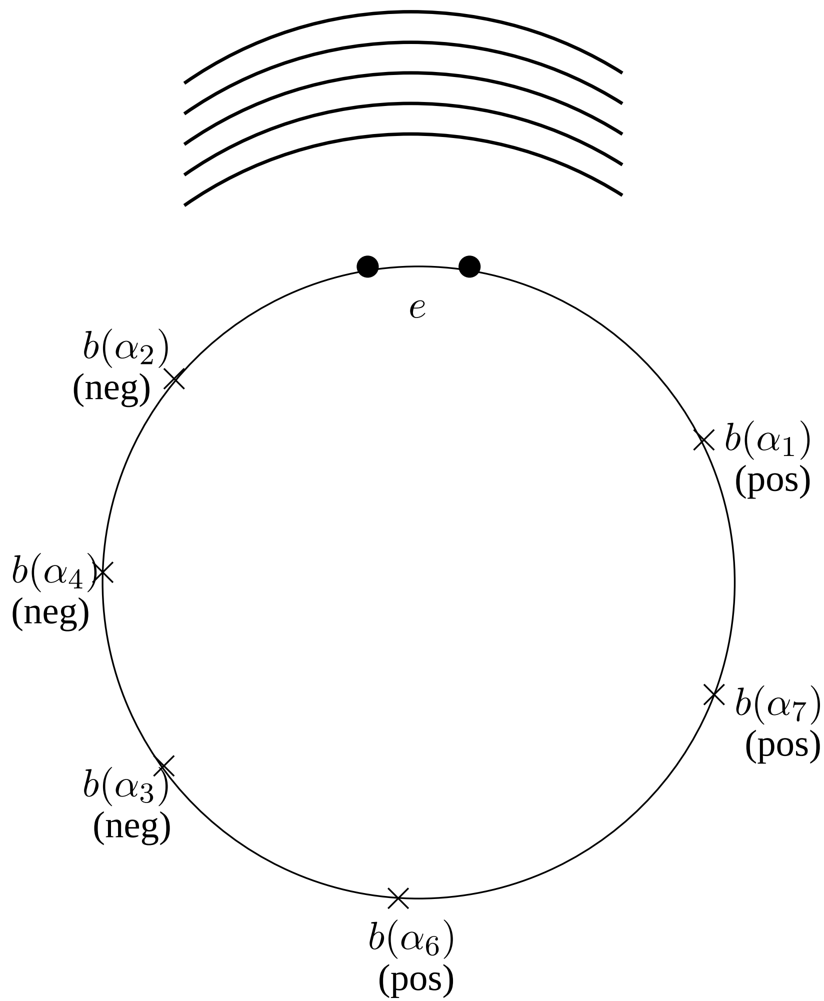

e
$\times b(\alpha_1)$
(pos)


e
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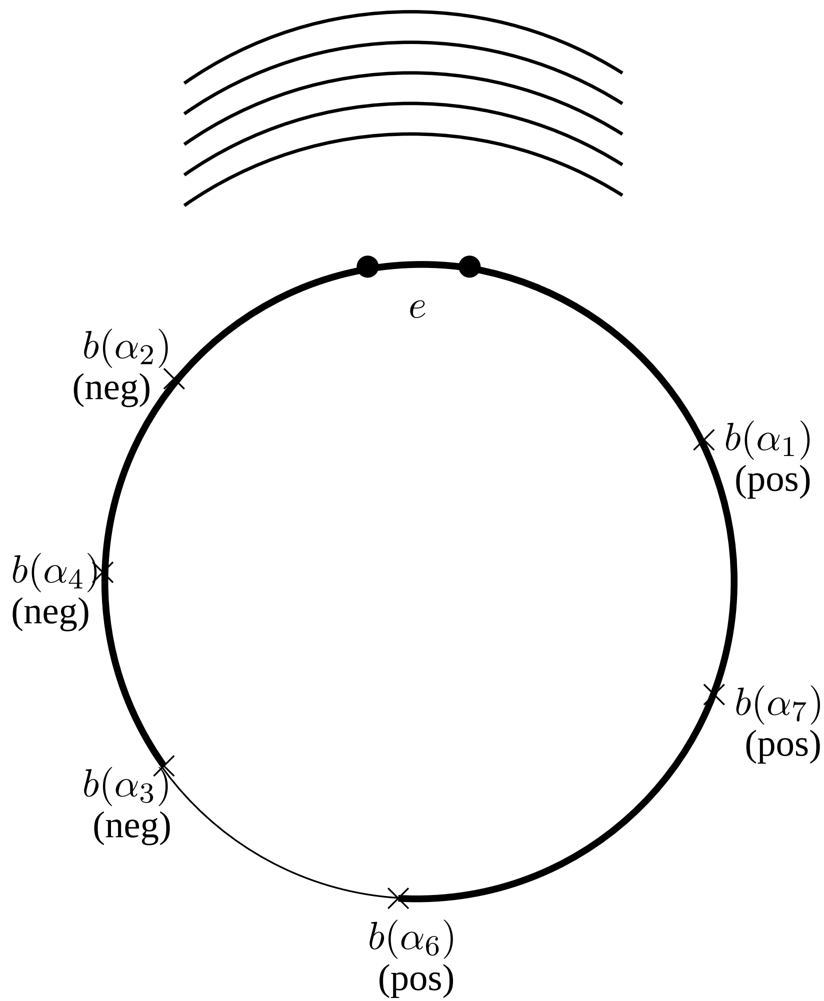
$b(\alpha_6)$
(pos)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(neg)
$b(\alpha_7)$
(pos)
$b(\alpha_3)$
(neg)
$b(\alpha_6)$
(pos)

Establishing an edge with high load

Repeated application of the previous Lemma yields:

Lem. In every S-PMC(RING) game $\langle G, \mathcal{P}, k\rangle$ with $\hat{\mu} \geq k$ there is an edge with load at least $\frac{\hat{\mu}k}{4}$

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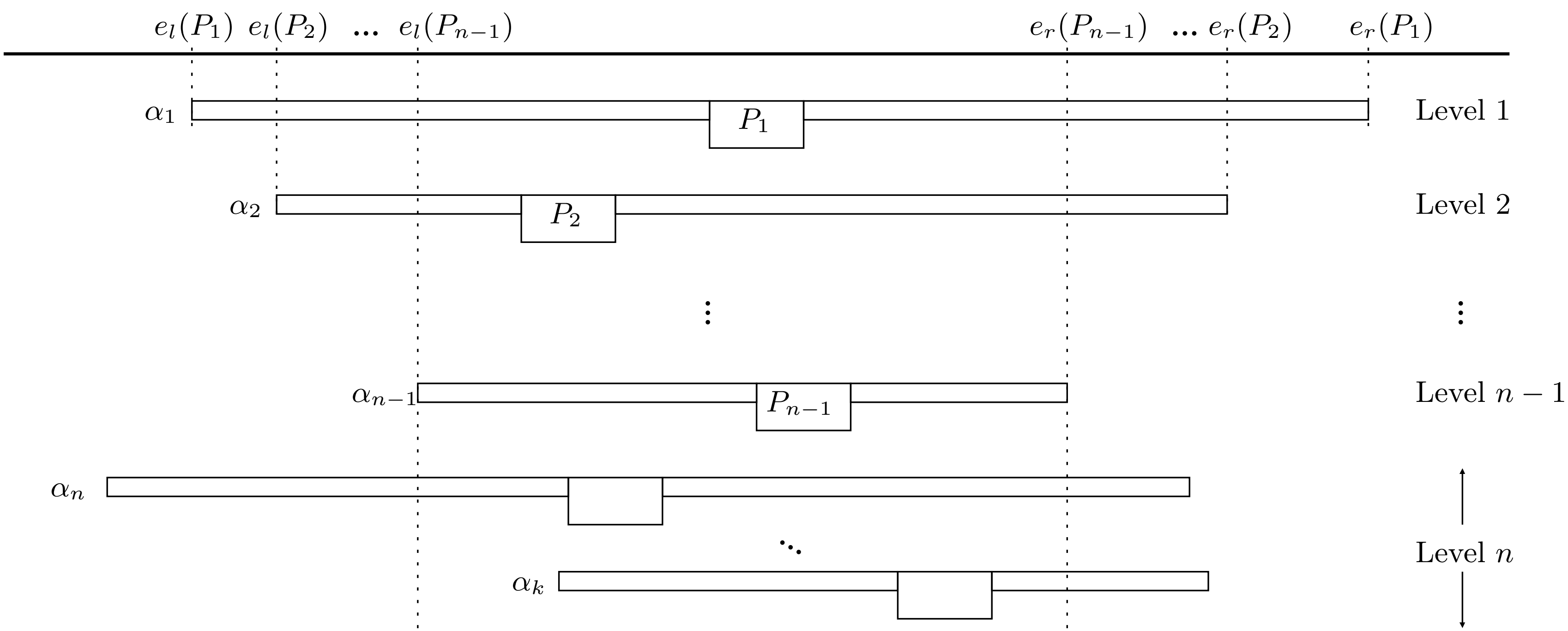

Constant PoA for $L = \Omega(k^2)$

Thm. For any S-PMC(RING: $L = \Omega(k^2)$) game, $\text{PoA} = \mathcal{O}(1)$

Proof.

- If $\hat{\mu} \geq k$, then $L \geq \frac{\hat{\mu} k}{4} \Rightarrow \mu_{\text{OPT}} \geq \frac{\hat{\mu}}{4} \Rightarrow \text{PoA} \leq 4$
- If $\hat{\mu} < k$, then:

$$\text{PoA} = \frac{\hat{\mu}}{\mu_{\text{OPT}}} \leq \frac{\hat{\mu} k}{L} < \frac{k^2}{L} = \mathcal{O}(1)$$

Unbounded PoA for $L = o(k^2)$

Thm. For any $\varepsilon > 0$ there is an infinite family of S-PMC(CHAIN: $L = \Theta(k^{2-\varepsilon})$) games with $\mathrm{PoA} = \Omega(k^{\frac{\varepsilon}{2}})$.

Colored resource allocation games

- Players have access to a set of facilities F, each one available in k different colors
- Each player i picks some facility combination from $\mathcal{E}_i \subseteq 2^F$
- Each player must pick facilities of the same color

The last constraint nicely models the wavelength continuity constraint

B, Pagourtzis, Pierrakos, Syrganis: Colored resource allocation games. CTW 2009.

Various player costs

Def Colored congestion games

$$C_i = \sum_{e \in E_i} n_{e,a_i}$$

Def Colored bottleneck games

$$C_i = \max_{e \in E_i} n_{e,a_i}$$

Various social costs

$$\mathrm{sc}_{\mathrm{max}} = \max_{i \in \mathcal{P}} C_i$$

$$\mathrm{sc}_{\mathrm{sum}} = \sum_{i \in \mathcal{P}} C_i$$

$$\mathrm{sc}_{\mathrm{fib}} = \sum_{e \in F} \max_{a \in W} n_{e,a}$$

Price of anarchy bounds

Social cost	Colored Congestion Games	Congestion Games
$\mathrm{SC}_{\max}$	$\Theta\left(\sqrt{\frac{N}{k}}\right)$	$\Theta\left(\sqrt{N}\right)$
$\mathrm{SC}_{\mathrm{sum}}$	$\frac{5}{2}$	$\frac{5}{2}$
$\mathrm{SC}_{\mathrm{fib}}$	$\Theta\left(\sqrt{k \cdot \lvert F \rvert}\right)$	---

Social cost	Colored Bottleneck Games	Bottleneck Games
$\mathrm{SC}_{\max}$	$\Theta\left(\frac{N}{k}\right)$	$\Theta\left(N\right)$
$\mathrm{SC}_{\mathrm{sum}}$	$\Theta\left(\frac{N}{k}\right)$	$\Theta\left(N\right)$
$\mathrm{SC}_{\mathrm{fib}}$	$\Theta\left(\frac{\lvert E_{\vec{A}} \rvert}{\lvert E_{A^*} \rvert} \frac{N}{k}\right)$	---

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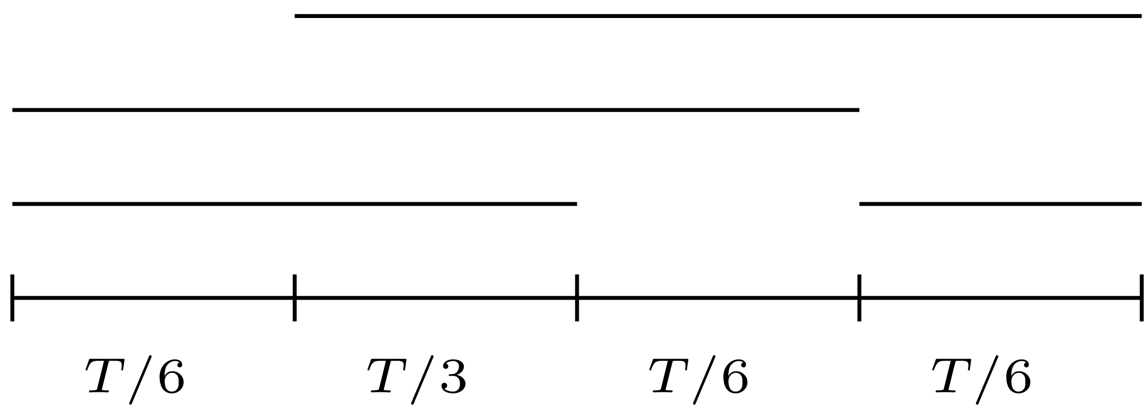
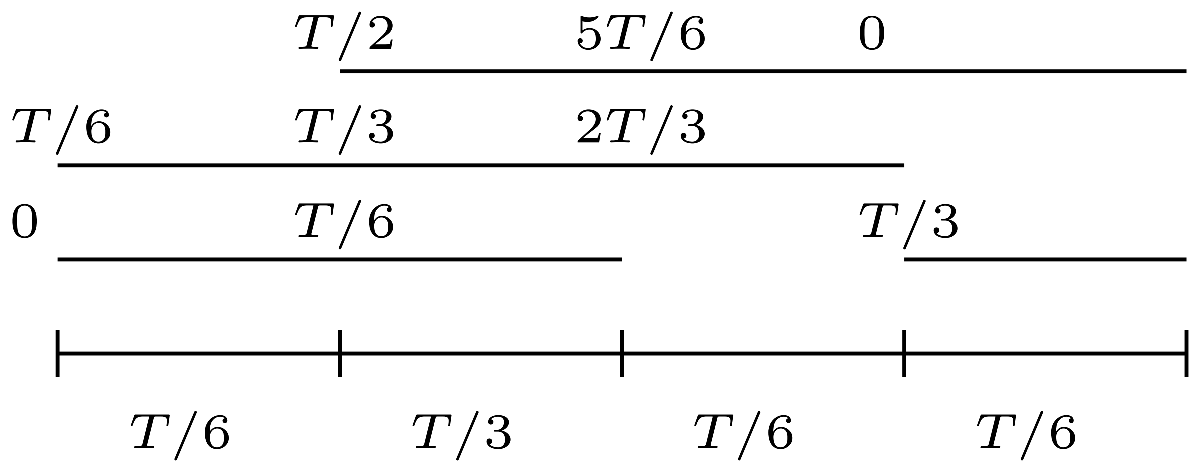
Headway maximization



B, Kaouri, Lampis, Pagourtzis: Periodic Metro Scheduling. ATMOS 2006.

Headway maximization



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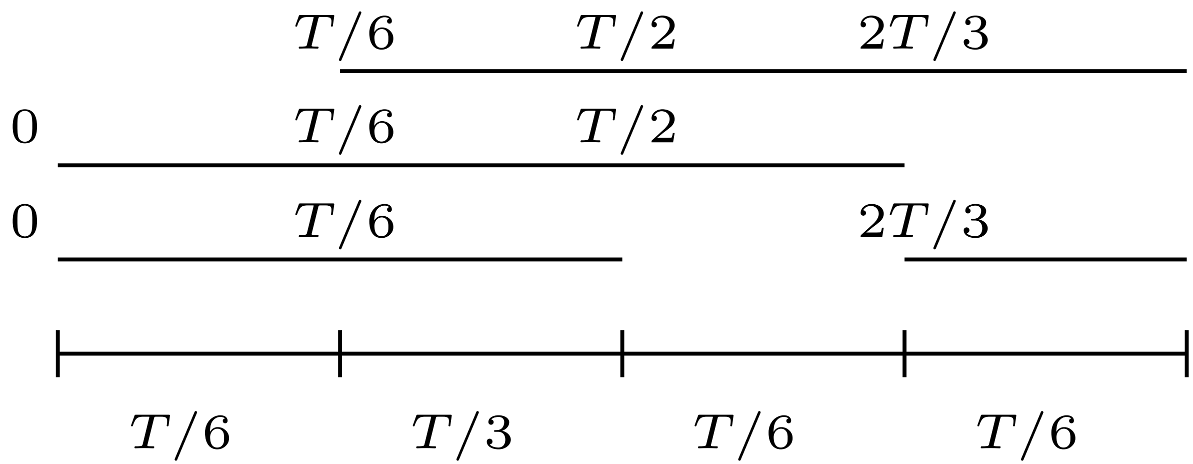

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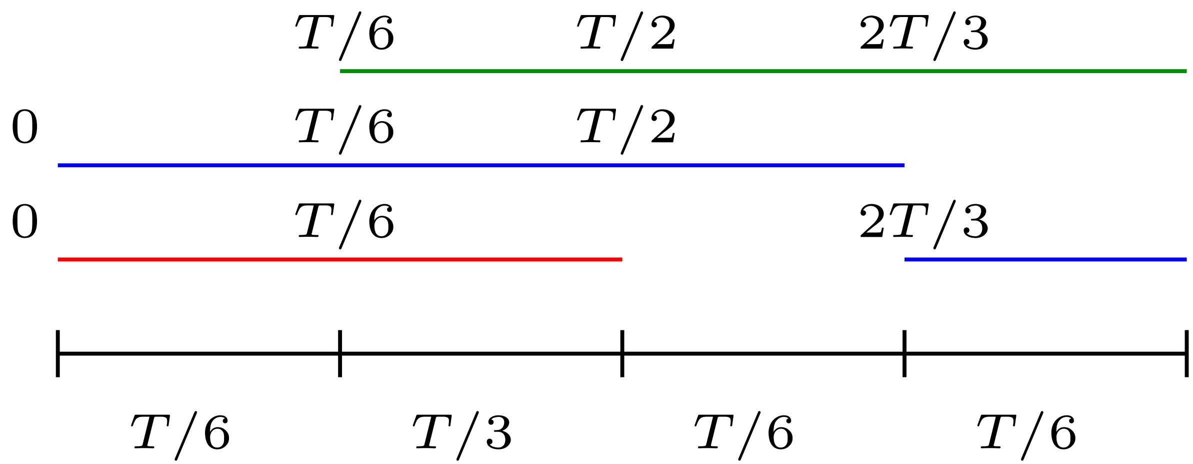
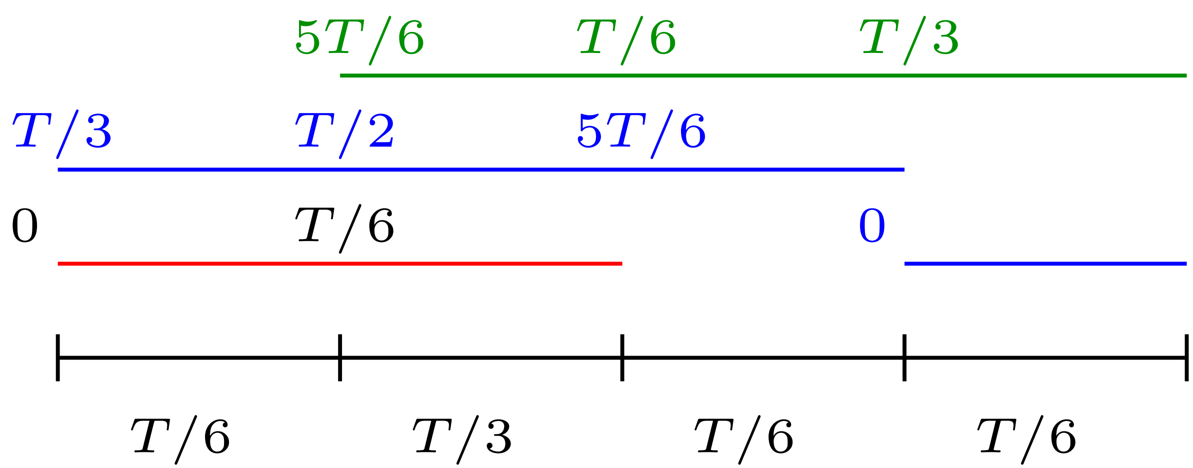


Headway maximization



Headway maximization (PMS)

Thm If a PMS instance admits a full collision schedule, then:

- A k-coloring yields a schedule with headway at least $\frac{T}{k}$
- A schedule with headway h yields a $\left\lceil \frac{T}{h} \right\rceil$-coloring

Thm If a PMS instance admits a full collision schedule, then a ρ-approximate coloring yields a $\left(\frac{1}{\rho} \cdot \frac{L}{L+1}\right)$-approximate schedule.

Conclusions

- Match and replace for MaxPr-PC in rings
- Selfish path multicoloring
- A framework for studying non-cooperative resource allocation in multifiber networks
- Applicability of path coloring models to a wide range of problems in networking/scheduling

Other publications

E. Bampas, L. Gąsieniec, R. Klasing, A. Kosowski, T. Radzik: Robustness of the rotor-router mechanism. OPODIS 2009 (to appear, LNCS).

E. Bampas, L. Gąsieniec, N. Hanusse, D. Ilcinkas, R. Klasing, A. Kosowski: Euler tour lock-in problem in the rotor-router model. DISC 2009 (to appear, LNCS vol. 5805).

E. Bampas, A. Goebel, A. Pagourtzis, A. Tentes: On the connection between interval size functions and path counting. TAMC 2009 (LNCS vol. 5532).

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E. Bampas, A. Goebel, A. Pagourtzis, A. Tentes: On the connection between interval size functions and path counting. TAMC 2009 (LNCS vol. 5532).

Thank You!